

TRIGONOMETRIC IDENTITIES

Fundamental Identities

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 & \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \tan \theta \cot \theta &= 1 & \cot \theta &= \frac{\cos \theta}{\sin \theta} \\ 1 + \tan^2 \theta &= \sec^2 \theta & \sec \theta &= \frac{1}{\cos \theta} \\ 1 + \cot^2 \theta &= \csc^2 \theta & \csc \theta &= \frac{1}{\sin \theta}\end{aligned}$$

Half Angle Identities

$$\begin{aligned}\sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} & \cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{\theta}{2} &= \frac{\sin \theta}{1 + \cos \theta} & \tan \frac{\theta}{2} &= \frac{1 - \cos \theta}{\sin \theta}\end{aligned}$$

Double Angle Identities

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta & \cos 2\theta &= 2 \cos^2 \theta - 1 \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta & \cos 2\theta &= 1 - 2 \sin^2 \theta \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} & \cot 2\theta &= \frac{\cot \theta - \tan \theta}{2}\end{aligned}$$

Triple Angle Identities

$$\begin{aligned}\sin 3\theta &= 3 \sin \theta - 4 \sin^3 \theta & \cos 3\theta &= 4 \cos^3 \theta - 3 \cos \theta \\ \tan 3\theta &= \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} & \cot 3\theta &= \frac{1 - 3 \tan^2 \theta}{3 \tan \theta - \tan^3 \theta}\end{aligned}$$

Addition/Subtraction Identities

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \sin \beta \cos \alpha \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}\end{aligned}$$

Power Reduction Identities

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

Inverse Identities

$$\sin(\sin^{-1} x) = x \quad \cos(\cos^{-1} x) = x$$

Symmetry Identities

$$\begin{aligned}\sin(-\theta) &= -\sin \theta & \cos(-\theta) &= \cos \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta \\ \sec(-\theta) &= \sec \theta & \csc(-\theta) &= -\csc \theta \\ \sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \\ \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan \theta \\ \sec\left(\frac{\pi}{2} - \theta\right) &= \csc \theta & \csc\left(\frac{\pi}{2} - \theta\right) &= \sec \theta \\ \sin(\pi - \theta) &= \sin \theta & \cos(\pi - \theta) &= -\cos \theta \\ \tan(\pi - \theta) &= -\tan \theta & \cot(\pi - \theta) &= -\cot \theta \\ \sec(\pi - \theta) &= -\sec \theta & \csc(\pi - \theta) &= \csc \theta\end{aligned}$$

Angle Shift Identities

$$\begin{aligned}\sin\left(\frac{\pi}{2} + \theta\right) &= \cos \theta & \cos\left(\frac{\pi}{2} + \theta\right) &= -\sin \theta \\ \tan\left(\frac{\pi}{2} + \theta\right) &= -\cot \theta & \cot\left(\frac{\pi}{2} + \theta\right) &= -\tan \theta \\ \sec\left(\frac{\pi}{2} + \theta\right) &= -\csc \theta & \csc\left(\frac{\pi}{2} + \theta\right) &= \sec \theta \\ \sin(\pi + \theta) &= -\sin \theta & \cos(\pi + \theta) &= -\cos \theta \\ \tan(\pi + \theta) &= \tan \theta & \cot(\pi + \theta) &= \cot \theta \\ \sec(\pi + \theta) &= -\sec \theta & \csc(\pi + \theta) &= -\csc \theta\end{aligned}$$

Product-to-Sum Identities

$$\begin{aligned}\sin \alpha \cos \beta &= \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] \\ \sin \alpha \sin \beta &= \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]\end{aligned}$$

Sum-to-Product Identities

$$\begin{aligned}\sin \alpha + \sin \beta &= 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \\ \sin \alpha - \sin \beta &= 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} \\ \cos \alpha + \cos \beta &= 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \\ \cos \alpha - \cos \beta &= -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}\end{aligned}$$